Challenging Question: Use the DELTA method to deduce Greenwood's formula

1. Remember the KM estimate $S_n(t) = \prod_{i:y_i \le t} \left(\frac{r_i - s_i}{r_i} \right)$ and the corresponding estimator $S_n(t) = \prod_{i:y_i \le t} \left(\frac{r_i - S_i}{r_i} \right)$. We will assume that y_i and r_i are known (not random).
2. Deduce the sampling distribution of S_i and show that $E(S_i) = r_i\, p_i$ and that $\text{var}(S_i) = r_i\, p_i\,(1 - p_i)$ where p_i is the probability of "dying" at time y_i assuming that the policyholder is "alive" immediately before y_i.
3. As it is difficult to deal with the variance of a product of random variables show that $\text{var}\left(\ln\left(S_n(t)\right)\right) = \sum_{i:y_i \le t} \text{var}\left(\ln\left(\frac{r_i - S_i}{r_i} \right)\right)$
4. Use the DELTA method to show that $\text{var}\left(\ln\left(\frac{r_i - S_i}{r_i} \right)\right) \simeq \frac{p_i}{r_i\,(1 - p_i)}$
5. Use the DELTA method again to show that $\text{var}\left(S_n(t)\right) \simeq S(t)^2\, \text{var}\left(\ln\left(S_n(t)\right)\right)$
6. Summarize the previous findings showing that $\text{var}\left(S_n(t)\right) \simeq S(t)^2 \sum_{i:y_i \le t} \frac{p_i}{r_i\,(1 - p_i)}$
7. Use the "natural" estimate of p_i, $\hat{p}_i = s_i / r_i$, to show that $\widehat{\text{var}}\left(S_n(t)\right) \simeq S_n(t)^2 \sum_{i:y_i \le t} \frac{s_i}{r_i\,(r_i - s_i)}$